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The provision point mechanism with refund bonuses

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Abstract

We introduce refund bonuses into the provision point mechanism. If a total contribution is less than the provision point, each contributor receives not only his contribution refunded but also a refund bonus the size of which is proportional to the contribution made. However, because of competition for refund bonuses the provision point is reached in equilibrium. Furthermore, the mechanism can uniquely implement the public good project with Lindahl prices. The mechanism also has other applications.

Keywords: Public goods, provision point mechanism, refund bonus, distributional efficiency *JEL:* D82, H41

1. Introduction

For private funding of discrete public goods, a frequently applied fund-raising method is the voluntary contribution mechanism with a provision point, commonly known as the provision point mechanism (Palfrey and Rosenthal (1984); Bagnoli and Lipman (1989); also see Andreoni (1998)). This method has a long history of applications, perhaps the most famous example of which is Joseph Pulitzer's fund-raising campaign for the construction of the pedestal for the Statue of Liberty in New York. Most recently, the mechanism has been successfully applied by Internet crowdfunding platforms such as Kickstarter and Indiegogo for funding numerous public projects ranging from open-air art exhibitions, skateboarding parks, preservation of archaeological sites to the launch of the first public space telescope.¹ The provision point mechanism owes its popularity to its simple structure in spite of the implementability concerns that this structure raises. In-

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¹For more on crowdfunding, see Young (2013).

deed, the mechanism is fraught with multiple equilibria, both efficient and inefficient, and particularly with free riding.² This paper offers a simple modification that significantly improves the mechanism's properties up to strict implementation.

As an illustration, consider a group of people that can benefit from a public good project, say, a \$1000 drinking fountain. Under the provision point mechanism with refunds, the drinking fountain is provided when at least \$1000 is raised in contributions, which are refunded otherwise. Obviously, the zero-contribution outcome is equilibrium as is any other combination of individually rational contributions that sum up to the provision point. Now imagine that one group member contributes \$100 and announces that if others contribute less than \$900 in total, then he will divide his contribution among others in the proportion of their individual contribution to the total contribution. Namely, in the event of insufficient contributions, each contributor gets his contribution back plus a share of \$100 as refund bonus. But with this modification, the only equilibrium outcome is the provision of the public good.

To see this, first observe that the zero-contribution outcome is not equilibrium. By just contributing a penny, any member could get the entire \$100 as refund bonus. With the refund bonus increasing in own contribution, no outcome with total contributions less than \$900 can make an equilibrium. The only possible equilibrium outcome is when total contributions exactly reach the provision point of \$900, the necessary condition for which is that the promised bonus money of \$100 does not exceed the net value of the public good. Thus, in equilibrium the public good is provided without the distribution of refund bonuses. The same outcome can also be achieved when the mechanism designer does not make any contribution, but sets the provision point at \$1000 and promises an amount of bonus money in the case of non-provision. In the paper, we analyze the latter version of the mechanism and also discuss its other variants.

Under the proposed mechanism, every consumer obtains an equilibrium payoff from

²In laboratory experiments, the success rate of the provision point mechanism is about 50 percent (see Isaac et al. (1989), Cadsby and Maynes (1999); for reviews, see Ledyard (1995) and Chen (2008)). In the field, it is significantly lower (Rose et al. (2002)). In 2013, Kickstarter reported the success rate of 44% for all of its initially pre-screened crowdfunding campaigns, which also included projects other than for public goods.

the public good at least as high as that from the highest refund bonus assigned to him if he deviates. Therefore, the effect of refund bonuses is the reduction of the set of strategies that can be supported in equilibrium. More generous refund bonuses imply a smaller set of equilibrium strategies as deviations become more profitable. With bonus money set at the net value of the project, the mechanism not only uniquely implements the public good project but does so with Lindahl pricing: Consumers contribute the same proportion of their valuations for the public good.

The introduction of refund bonuses into the provision point mechanism resolves the equilibrium coordination problem. For the same reason, a similar mechanism can be applied to other problems with multiple Pareto-ranked equilibria such as the collective action problem or markets with adverse selection (Akerlof (1970)). In these problems, schemes with bonus money can be designed so that they eliminate undesirable equilibria leaving only the efficient ones, which, by design, do not lead to the distribution of refund bonuses. From this more general perspective, our mechanism can be viewed as a practical application of the augmented revelation principle of Mookherjee and Reichelstein (1990), where side payments are designed to eliminate non-truthful equilibria.

In the next section, we discuss related literature. In Section 3, we introduce the provision point mechanism with bonus money and analyze its performance under complete information. In Section 4, we discuss sources of bonus money, different informational environments, present a class of equivalent mechanisms, and also discuss other applications of the mechanism. The last section concludes the study.

2. Related literature

In social cooperation dilemmas, rewards play an important role in inducing higher levels of cooperation. For funding public goods, a well known example of a mechanism with rewards is a lottery.³ Morgan (2000) studies a lottery mechanism where a fixed amount from ticket revenues is used for lottery prizes with the rest of the revenues spent

³For other examples, see Falkinger (1996) who proposes a mechanism that rewards contributors with above-average contributions. Goeree et al. (2005) demonstrate the advantages of the all-pay auction design in soliciting contributions.

on public goods. He demonstrates efficiency gains of the lottery mechanism over the voluntary contribution mechanism. Off the equilibrium path, there is a close connection between the lottery mechanism studied in Morgan (2000) and the mechanism with refund bonuses proposed here. The lottery mechanism also has inefficient equilibria of low contributions unless the lottery organizer has a budget to fill the difference between the ticket revenues and the promised lottery prize so that the lottery is not recalled. This clause eliminates low-contribution equilibria as do refund bonuses in our mechanism. However, even though in the lottery mechanism rewards increase allocative efficiency, they may impair distributional efficiency as poorer people, motivated by lottery prizes, end up contributing disproportionately too much.

In the event of insufficient contributions, the idea to reward contributors also appears in Tabarrok (1998).⁴ In his model, agents have a binary choice of making or not making a pre-determined contribution toward a public good, which is provided conditional on a sufficient number of contributors. He proposes an “assurance contract” that specifies a reward that each contributor receives in case the number of contributors misses the target needed for implementation. With such a reward, like in the present paper, the mechanism designer can effectively and at no cost eliminate inefficient outcomes. The present paper is a generalization of this idea both in terms of strategy spaces and other applications. The main advantage of our mechanism with continuous contributions lies in its superior properties of distributional efficiency. For a similar reason, our mechanism can also achieve allocative efficiency in environments with very uneven distributions of private valuations, where an assurance contract with pre-determined contributions may be restrictive in raising sufficient funds.

Another strand of literature emphasizes the role of punishments for inducing contributions toward public goods. In environments where agents are perfectly informed about each other, Varian (1994) proposes a mechanism with punishments that are imposed on agents if their reported own Lindahl price differs from what other agents report as their Lindahl price. The punishment structure ensures truthful reports and, consequently, the

⁴I thank Ted Bergstrom for bringing my attention to this work.

implementation of the Lindahl allocation. Even though Andreoni and Varian (1999) demonstrate the effectiveness of the mechanism in laboratory experiments, the problem with the mechanism is that it requires the authority to punish and is rather complex. Our proposed mechanism achieves the same outcome but through an incentive structure in reverse of that in Varian (1994). Namely, each agent reports his own Lindahl price and is rewarded if someone else shades his own, but no authority is required for implementation.

3. The provision point mechanism with bonus money

There are a set $N = \{1, \dots, n\}$ of consumers and a discrete public good, which costs C to provide. A consumer i 's willingness to pay for the public good is given by $v_i \geq 0$, $i \in N$, to which we also refer as his valuation. Until further notice, we assume that individual valuations are publicly known. Let V denote the sum of consumers' valuations.

A mechanism designer solicits voluntary contributions toward the public good. Let g_i denote consumer i 's contribution and G the sum of contributions. If $G \geq C$, the public good is financed out of the contributions collected, with the excess amount $G - C$ wasted (assumed for the ease of exposition). If $G < C$, the public good is not provided, the contributions are refunded, but also each contributor receives a refund bonus $\frac{g_i}{G}R$, where R is the amount of bonus money promised by the mechanism designer from own budget in the beginning of the campaign. The payoff to consumer i is given by

$$\pi_i(g_i, G) = \begin{cases} \mathcal{I}(G \geq C)[v_i - g_i] + \mathcal{I}(G < C) \left[\frac{g_i}{G}R \right] & \text{if } G > 0 \\ 0 & \text{if } G = 0, \end{cases} \quad (1)$$

where $\mathcal{I}(\cdot)$ is an index function.

We assume that consumers choose contributions (without randomizing) that constitute a Nash equilibrium of the game induced by mechanism R , which is short for a mechanism with promised bonus money R . Letting G_{-i} denote the sum of all contributions of consumers other than i , we define

Definition 1. A vector of contributions (g_i^*) , $i = 1, \dots, n$, is a Nash equilibrium if for each i , g_i^* maximizes $\pi_i(g_i, G_{-i}^* + g_i)$.

The next proposition characterizes the set of pure-strategy Nash equilibria, which we denote by $\Gamma(R)$.

Proposition 1. *Let $V > C$ and $R > 0$. $\Gamma(R) = \{(g_i^*) : \forall i, g_i^* \leq \frac{C}{R+C}v_i, G^* = C\}$ if $R \leq V - C$. Otherwise, $\Gamma(R) = \{\emptyset\}$.*

Proof. In equilibrium, $G^* < C$ cannot hold as any consumer could obtain a higher refund bonus by marginally increasing his contribution because of $R > 0$. Likewise, any consumer with a positive contribution could gain in utility by marginally decreasing his contribution if $G^* > C$. Thus, the equilibrium candidates need to have $G^* = C$. A vector (g_i^*) is an equilibrium if for each consumer i the net utility from the public good, $v_i - g_i^*$, exceeds the highest possible refund bonus, $\frac{g_i^*}{C}R$, or after transformations

$$g_i^* \leq \frac{C}{R+C}v_i. \quad (2)$$

Summing up (2) yields $V - C \geq R$, which is the condition for equilibrium existence as it ensures that there is a vector (g_i^*) such that (2) is satisfied for all i . If $R > V - C$, then $G^* = C$ cannot hold as (2) will be violated at least for some i , implying $\Gamma(R) = \{\emptyset\}$. ■

According to Proposition 1, rewarding insufficient contributions induces an amount of contributions sufficient for implementation, the condition for which is that bonus money does not exceed the net utility from the public good. Put differently, in the game induced by the mechanism the consumers need to decide which “prize” to take, the bonus money R or the net utility of the public good $V - C$. Therefore, if $V - C \geq R$, they choose the public good and otherwise the bonus money. In the latter scenario, there is no fixed point because the action space is continuous and the upper semicontinuity of at least one payoff function π_i is violated. With discrete contributions, $R > V - C$ would result in the equilibrium outcome $G^* = C - \delta$, where δ is the smallest currency unit.

As a direct consequence of Proposition 1, we can show

Corollary 1. *If $R' > R$, then $\Gamma(R') \subseteq \Gamma(R)$.*

Proof. If (2) holds for R' it also holds for $R < R'$. But the reverse is not true. ■

Intuitively, in equilibrium each consumer has to obtain a net utility level from the public good at least as high as that from the refund bonus if he deviates. A higher level of reward money implies a more profitable deviation and, in turn, a higher level of utility for each consumer in equilibrium, which reduces the set of equilibria.

Finally, when bonus money is set at the net value, $R = V - C$, the mechanism implements the public good as the unique equilibrium.⁵ This equilibrium has a special feature. All consumers contribute the same proportion of their valuations, i.e., $g_i = \frac{C}{V}v_i$. Thus, the Lindahl tax of C/V on consumers' valuations is effectively levied.

4. Discussion

4.1. Sources of bonus money

With a restriction to efficient projects, bonus money can be set so that it is not distributed in equilibrium, which means that the mechanism designer needs only to possess a credible capacity of raising the money promised. One can think of various potential sources of bonus money, the simplest of which is seed money generated from individual donors. Thinking of crowdfunding, a possible source could be an insurance fund raised from premia paid by successful campaigners. An interesting example is when the designer is a tax authority applying the mechanism that has a capacity of raising bonus money from taxes levied ex post. Below, we discuss this example in more detail.

The authority applies a mechanism R , which, when needed, is raised from lump-sum taxes of $\tau = R/n$ levied from every consumer. (Assume that the maximum tax revenue that the authority can generate is less than C .) Ex-post taxation implies a change in consumer payoffs as the second term in (1) becomes $\mathcal{I}(g_i + G_{-i} < G) \left[\frac{g_i R}{g_i + G_{-i}} - \tau \right]$. Now, analogously to (2), an equilibrium contribution has to satisfy

$$g_i^* \leq \frac{C}{R + C}(v_i + \tau). \quad (3)$$

⁵Formally, the uniqueness result can be explained by the observation that at $G = C$ the payoff function π_i of each consumer i is continuous in g_i when $R = V - C$. At the same time, we have a multiplicity of equilibria with $R < V - C$ because of the discontinuity of payoff functions π_i , which are however upper semicontinuous. But if $R > V - C$, then at $G = C$ the upper semi-continuity of at least one payoff function π_i is violated.

Thus, the upper bound on equilibrium contributions increases when the bonus money R is raised by ex-post taxes. The reason is that, when the provision point is reached, the consumer also avoids paying the tax, which makes his gain from the public good $v_i + \tau$ rather than v_i . Summing up (3) we obtain the equilibrium condition $V \geq C$. As long as the public good is efficient, consumers coordinate on its provision rather than on obtaining the bonus money, the total utility of which is reduced to zero because of taxes.

Besides sources of bonus money, the question about its level is also empirically important. While the theory predicts the same aggregate outcome no matter how little bonus money is promised, in practice its level is likely to matter, e.g., “nuisance costs” involved need to be covered at the very least.

4.2. Aggregative game and incomplete information

The game induced by mechanism R is aggregative: A consumer’s payoff depends on own contribution and the sum of all contributions. Due to the aggregative structure, the game can be analyzed as a two-player game between an individual consumer and the “aggregate” of other consumers. An immediate consequence of this observation is that we can relax the complete information assumption. Proposition 1 and its proof continue to hold when it is only the first moment of the empirical distribution of private valuations that is publicly known. This informational structure implies that there is no aggregate uncertainty about the total value of the public good, which in turn implies complete information in the two-player representation of the aggregative game in question.

Proposition 1 also follows from the aggregate concurrence principle in aggregative games: In equilibrium, agents must agree on the choice of aggregate outcomes (Martimort and Stole (2011, 2012)). As it immediately follows from this principle, the equilibria produced by our mechanism are coalition-proof. This principle can also be applied to the case with incomplete information without imposing any restrictions on the sum of private valuations, which is then a random variable. The provision of the public good remains the unique equilibrium outcome when $C + R \leq \inf \mathcal{V}$, where \mathcal{V} is the support of the sum of valuations; otherwise, the equilibrium does not exist for similar reasons to the

ones discussed after Proposition 1.⁶ But if we allow for a smallest currency unit, then in equilibrium the public good is provided with probability $1 - F(C + R)$, where F is the distribution of the sum of valuations.

4.3. Refund bonus rules

It is the monotonicity of refund bonus in own contribution that is behind the unique equilibrium outcome in Proposition 1, not its particular form. Therefore, other provision point mechanisms with increasing refund bonuses could also lead to the same result. For example, consider a refund bonus rule which is a fixed share of own contribution, i.e., in the event of insufficient contributions each contributor gets his contribution refunded plus x % of his contribution as a bonus. Clearly, a mechanism with this rule is equivalent in equilibrium outcomes, both in aggregate and individual levels, to the mechanism $R = xC$. More generally, as one can easily show, an aggregate invariant mechanism⁷ that has increasing refund bonuses and pays in bonuses at most R in the limit is equivalent in aggregate equilibrium outcomes to mechanism R .

The observation that efficiency can be achieved by applying different refund bonus rules is of relevance with regard to redistribution. As individual contributions and, ultimately, levels of private of consumption depend on the refund bonus rule applied, the mechanism designer can also attempt pursuing various redistributive goals by designing specific bonus rules. Arguably, a wider range of redistributive goals becomes feasible if personalized bonus rules are allowed.

4.4. Other applications

For the provision of public goods, as we have shown, the introduction of refund bonuses can effectively resolve the equilibrium coordination problem. Similar mechanisms can be applied to other situations where the problem of equilibrium coordination arises. Consider a collective action problem, where participation is individually rational only when a critical mass of participants is reached so that there are two equilibria of low and of high

⁶For details, see companion working paper Zubrickas (2013).

⁷I.e., in the game induced by the mechanism total utility to consumers depends only on their total contribution.

participation (e.g., committee voting with quorum). Designing rewards for participants in the event of low participation can implement the desirable high-participation outcome without actually distributing any rewards.

Arguably, the mechanism with bonus money can also be applied to markets with adverse selection that have multiple equilibria. As an illustration, consider a competitive labor market with privately known productivities that has two locally stable equilibria of low and of high employment (as studied, e.g., in Mas-Colell et al. (1995, Ch. 13B)). Let the labor market be in the low-employment equilibrium. The government announces the following stimulus package: Firms will divide a certain (sufficiently high) amount of stimulus money in proportion to their newly created jobs if the total number of jobs created is less than a certain threshold. If the threshold is set where the expected worker productivity exceeds the wage paid, the total number of jobs created will surpass the threshold and, ultimately, reach the level of the high-employment equilibrium.

5. Conclusion

Despite advances in the development of efficient mechanisms, it is the simple provision point mechanism with very weak implementation properties that has received a lot of practical attention for the private provision of discrete public goods. In this paper, we suggest a modification to the provision point mechanism that significantly improves its properties. The idea is to introduce refund bonuses that are paid in the event of insufficient contributions. If the refund bonus increases in own contribution, then competition for refund bonuses and preference for the public good induce contributions up to the level where the public good is provided. A mechanism with refund bonuses can also be designed to implement uniquely the public good project with Lindahl prices. From a normative perspective, a mechanism with refund bonuses ensures that contributors always receive a positive utility, whereas free riders may end up with nothing. Finally, mechanisms with bonus money can be designed for other situations where the problem of equilibrium coordination arises.

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